5 Application of Scaling to the Analysis of Unstable Flow Phenomena

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Water infiltration into layered soils where a fine-textured soil overlays a coarse sand exhibits Taylor instability, and the flow field in the coarse sand breaks into fingers. Following the pioneering work of Saffman and Taylor (1958), Hill and Parlange (1972) studied this phenomenon experimentally and later proposed a theoretical model (Parlange & Hill, 1976). In this first experiment, heterogeneous packing of the coarse sand promoted merging of fingers resulting in a small number of near-saturated fingers. The increased water content resulting from merging of fingers is readily apparent from the experiments of Glass et al. (1989a). The theory provided a good description of finger diameters both in the laboratory and the field (Starr et al., 1978). Later, Hillel and Baker (1988) pointed out that when water first enters the coarse sand the matric potential should be expected to be negative and the finger to be unsaturated. The theory of Parlange and Hill (1976) applies equally well to that case, yielding for average finger width, D,

$$D = \left[\frac{\pi S_F^2}{K_F(\theta_F - \theta_i)}\right]\left[\frac{1}{1 - (q/K_F)}\right] \qquad [1]$$

Here, θ_i is the initial water content in the coarse sand, θ_F is the average water content in the fingers, and K_F and S_F are the corresponding conductivity and sorptivity. The experimental conditions for which Eq. [1]

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holds consists of a two-dimensional chamber (where the fingers can be conveniently observed) and a constant flux q entering the chamber per unit width of the chamber. The flux is controlled by the conductivity of the fine-textured soil layer overlaying the coarse sand.

We wrote Eq. [1] as the product of two square brackets. The first one is soil-dependent and requires a knowledge of θ_F. Even with a very homogeneous soil, Glass et al. (1989a) find that mergers take place between fingers. For instance, in their Fig. 7 each finger resulted from two mergers on the average. As mentioned earlier the water content in each individual finger is primarily a function of mergers, as well as the water entry potential in the coarse sand as discussed by Hillel and Baker (1988). Figure 7 of Glass et al. (1989a) also shows that there is a slight widening of fingers with water content, i.e., with the number of mergers. This is quite consistent with the form of the first bracket in Eq. [1] and the behavior of real soils. For a real soil S_F^2 increases more rapidly than $K_F(\theta_F - \theta_i)$ as θ_F increases toward saturation. There is, however, a compensation between the two factors so that the first bracket is not too sensitive to the exact value of θ_F, as observed experimentally. This is also quite useful in the interpretation of experiments with the help of Eq. [1]; i.e., even though the number of mergers will not be identical for all fingers, it will result in only slight variations in width from finger to finger so that the average width for one experiment remains quite representative of all the fingers in that experiment.

The second bracket in Eq. [1] depends on the ratio q/K_F, i.e., depends on the boundary condition through q and the soil through K_F. As discussed by Hillel and Baker (1988) the term q/K_F is also the fraction of the soil occupied by fingers. Thus, a convenient way to check the validity of Eq. [1] is to plot D as a function of the fraction of soil wetted by fingers. Figure 5–1 shows the dependence of D on the fractional area occupied by fingers for a particular sand. Experimental points (◇) are from Glass et al. (1989b) for sand with a mean diameter of 0.0991 cm and the theory (solid line) represents Eq. [1] with the first bracket estimated for that particular sand. For this sand, $D^* = D$. Notice that according to Eq. [1], finger diameter for two sands is not equal for the same wetted area. It is clear that there is a remarkable agreement between theory and observations.

We are now in position to check the application of the Miller and Miller (1956) theory of scaling to finger instability. With this theory, the scaled, or reduced, finger diameter should be independent of the sand type.

MILLER SCALING

In a fundamental paper, Miller and Miller (1956) discussed the relationship between properties for similar soils, i.e., soils packed identically and with identical particle distributions when scaled with the average particle size.

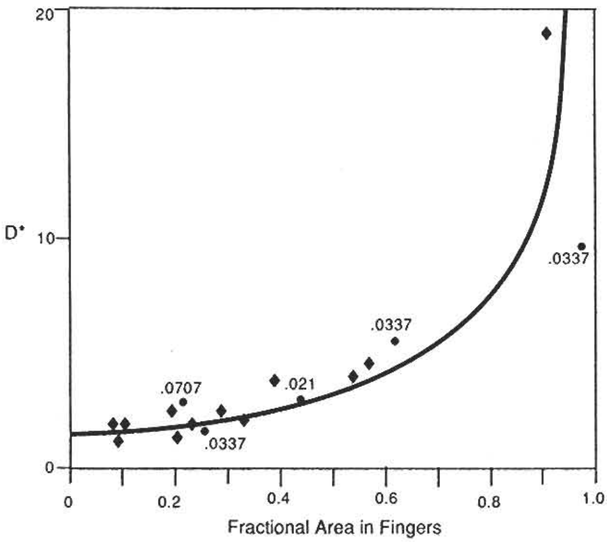

Fig. 5–1. Reduced finger width, D^*, as a function of the fractional area of wetted sand, i.e., occupied by the fingers. The diamonds correspond to $M = 0.0991$ cm, and the dots to $M = 0.0707$, 0.0337, and 0.021 cm as indicated on the figure. The solid line is the theoretical result obtained from Eq. [1] and [2].

To obtain such sands, a white silica sand used commercially for sand blasting was dry-sieved, yielding 10 sands of different mean grain sizes. Out of those, three were selected having similar particle size distributions as the sand used by Glass et al. (1989b). The average particle sizes, M, are 0.021, 0.0337, and 0.0707 cm, respectively, with the original one (Glass et al., 1989b) being $M = 0.0991$ cm. The maximum and minimum particle sizes are fairly close to the mean in each case with a distribution essentially uniform in between. Thus, the similarity of all distribution results if the spread of particle size divided by the average size is the same for all four sands. The numbers given in Table 5–1 show that this is indeed the case with a fairly good accuracy.

Under those conditions we expect the widths of the fingers to differ according to the first bracket of Eq. [1]. To calculate the dependence of this bracket on M we follow the rules for similar soils (Miller & Miller,

Table 5–1. Average particle size and relative size spread for four sands.

Mean grain diam. M	$\dfrac{\text{Maximum-minimum size}}{M}$
cm	
0.0991	0.330
0.0707	0.348
0.0337	0.365
0.0210	0.348

1956). The soil-water potential (pressure) Ψ, in the fingers behaves like M^{-1}, but the corresponding, θ_F, is the same. By analogy with Poiseuille flow K_F must behave like M^2, where as $S^2 \sim \int K d\Psi$ should behave like M. Altogether then the first bracket is proportional to M^{-1}. That is, the finger width is inversely proportional to the mean grain size. To take advantage of this result, we define a reduced finger width, D^* given by

$$D^* = D0.0991/M \qquad [2]$$

With this definition, for the same fractional area occupied by the fingers, all reduced finger widths, D^*, would be the same, and correspond to the physical width D when $M = 0.0991$ already plotted in Fig. 5–1.

Several experiments are reported in Fig. 5–1. It is clear that the three experiments with $M = 0.021$, 0.0337, and 0.0707 cm, respectively, when the fractional area occupied by fingers is less than 0.5 follows the reduction procedure very well. Two experiments when it is greater than 0.5 and for $M = 0.0337$ cm are also reported. The discrepancies for those two experiments are easily explained. As the fractional area increases, fingers are close to each other and some may coalesce, i.e., run parallel without actually merging. If those siamese twins are incorrectly counted as one, the result is an apparent finger width, which is greater than it should be. This is what happened when the fractional area was about 0.61. For the last experiment, q was close to the saturated conductivity so that the flow was close to being stable. In fact, the predicted finger width was several times larger than the chamber width. Under those conditions fingers could not develop. The spurious D^* indicated represents the wave length of the wavy front, which appeared in the chamber.

These last two results were given to point out that some care must be given in the interpretation of unstable flows. Luckily in practical situations, when fingers are observed in the field, the fractional area occupied by fingers is always small and those difficulties do not appear. In fact, for those field experiments the influence of the second bracket is always small.

In conclusion we have shown that Miller's scaling model can be applied with confidence to predict finger width for unstable flows. This should prove very useful in practice to easily estimate finger width in soils of a given texture and conclude whether instability could become a problem, for instance to predict the fate of contaminants.

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